QUANTUM GRAPHS - TCC COURSE - JANUARY-MARCH 2012

HOMEWORK 6

Playing with the tetrahedron

(1) Consider the tetrahedron graph, i.e. the complete Neumann graph with $V = 4$ vertices.
   (a) Assume that all bond lengths are incommensurate and obtain the length spectrum σ_ℓ up to the length of the graph, i.e. $\ell \leq \mathcal{L}$. What are the corresponding quantum amplitudes A_ℓ?
   (b) Now assume that all edge lengths are equal $l_b = \mathcal{L}/E$. How does the length spectrum and the corresponding amplitudes change?
   (c) Try to obtain the connectivity and the bond lengths back from the lower part of the length spectrum in the two cases considered above.

(2) Continue considering the tetrahedron, but this time as a discrete graph.
Assume that the number of orbits (not necessarily periodic) between i and j of length n is exponential in n for large n. Namely assume that $[C^n]_{ji} \sim ce^{\alpha n}$ for large n (where C is the connectivity matrix). Calculate α.
Hint: diagonalise C.